TRENTO, 2022/23
ADVANCED GROUP THEORY
SEMINAR PROPOSALS

Legenda. Seminars in red are already taken. Seminars in blue are partly taken, but there could be room for another student or sometimes two.

1. Nilpotent groups

These are dealt with in all introductory texts of group theory. There are some basic definitions and results in the notes.

2. Krull-Remak-Schmidt

See [Rob96], p. 80 ff.

3. Geometric interpretations of the Hall-Witt identity

See the references in the notes, in the section on nilpotent groups, in particular the article [Tao12] by Fields Medallist Terence Tao, and [Cal11].

4. The transfer

This is related to induced characters, see [Rob96, Ch. 10] or [Ser16, Ch. 7] or [Hup67] under *Verlagerung*.

5. Basic structure theory of rings

In the course, we prove that the complex group algebra $\mathbf{C}[G]$ of a finite group is isomorphic to a direct sum of matrix rings over $\mathbf{C}$. This is a special case of the basic structure theory of rings , which can be the argument for at least three different seminars.

One can check it in two books by Nathan Jacobson, who made essential contributions to this theory:

(1) [Jac64], first few chapters;
(2) [Jac89, Chapter 4].

6. Representation theory of compact groups

Compact groups admit a representation theory that is somewhat similar to that of finite groups. The point is that a compact group G admits a G-invariant measure, and the integral with respect to this measure replaces the sums typical of the representation theory of finite groups. This is material for at least a couple of seminars. One might see for instance [BtD85], and [Ser78]/[Ser77].

7. Recovering a representation from its character

See [Spe10].

8. Fourier analysis on finite abelian groups

The representation theory of finite abelian groups leads to the Discrete Fourier Transform, which you can read in [Ter99]. Note the telltale sign of a Vandermonde matrix (i.e., the character table of a cyclic group) in the Wikipedia article [DFT].

9. Tensor products of modules over non-commutative rings, and induced characters

Part of this I will cover in the lectures, and I have written something in the notes. The seminar would cover the case of (bi)modules over non-commutative rings such as the groups algebra, and its links to induced representations (which I will deal with in the lectures in a different way). See for instance [Lan02] for the tensor products.

References

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[Hup67] B. Huppert, *Endliche Gruppen. I*, Die Grundlehren der Mathematischen Wissenschaften, Band 134, Springer-Verlag, Berlin-New York, 1967. MR 0224703 (37 #302)

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[Rob96] Derek J. S. Robinson, *A course in the theory of groups*, second ed., Graduate Texts in Mathematics, vol. 80, Springer-Verlag, New York, 1996. MR 1357169 (96f:20001)

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[Ser78] ———, *Représentations linéaires des groupes finis*, revised ed., Hermann, Paris, 1978. MR 543841

[Ser16] ———, *Finite groups: an introduction*, Surveys of Modern Mathematics, vol. 10, International Press, Somerville, MA; Higher Education Press, Beijing, 2016, With assistance in translation provided by Garving K. Luli and Pin Yu. MR 3469786

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