TRENTO, 2020/21
ADVANCED GROUP THEORY
EXERCISE SHEET # 11

Exercise 11.1. Define induced modules.

Exercise 11.2. Define induced characters, and prove Frobenius reciprocity.

Exercise 11.3. Let G be a finite group. Let ρ be an irreducible representation of G, and χ be its character. Let $g \in G$. Prove that

(1) $\left| g^G \right| \chi(g)/\chi(1)$ is an algebraic integer.

Suppose now $\gcd(\left| g^G \right|, \chi(1)) = 1$. Prove that

(2) $\chi(g)/\chi(1)$ is an algebraic integer;
(3) if $\chi(g) \neq 0$, then $\rho(g)$ is a scalar matrix.

Exercise 11.4. Prove Burnside's theorem about the solubility of finite groups whose order is divisible by at most two distinct primes.