

**TRENTO, 2020/21**  
**ADVANCED GROUP THEORY**  
**EXERCISE SHEET # 7**

*Exercise 7.1.* Let  $G$  be a finite abelian group.

- (1) Show that all irreducible characters of  $G$  are linear.
- (2) Describe the irreducible characters of  $G$ .
- (3) Show that the dual group of  $G$  is isomorphic to  $G$ .

*Exercise 7.2.* Let  $G$  be a finite group,  $N \trianglelefteq G$ , and  $\pi : G \rightarrow G/N$  the natural homomorphism.

Let  $\rho : G/N \rightarrow \text{GL}(V)$  be a representation of  $G/N$ .

Show that  $\rho \circ \pi$  is a representation of  $G$ .

*Exercise 7.3.* Let  $G$  be a finite group. Show that there is a correspondence between

- (1) the linear characters of  $G$ , and
- (2) the linear characters of the abelian group  $G/G'$ .

*Exercise 7.4.* Construct the character table of  $S_3$ , and for each irreducible character construct a representation that affords it.