TRENTO, 2020/21
ADVANCED GROUP THEORY
EXERCISE SHEET # 6

Exercise 6.1.

(1) Show that the number of irreducible characters (which is the same as the number of pairwise irreducible representations) of a finite group G equals the number of conjugacy classes of G.
(2) Show that the character of the finite group G are class functions on G, and are a basis of the vector space of class functions $G \to \mathbf{C}$.

Exercise 6.2.

(1) Define the character table of a group.
(2) Show that a character table is an invertibile matrix.
(HINT: Use the orthogonality relations.)

Exercise 6.3. Describe the character table of a finite cyclic group.

Exercise 6.4. Recall that a character χ is called *linear* if $\chi(1) = 1$.

(1) Prove that a linear character is irreducible.
(2) Exhibit an example of a group G and an irreducible character of G which is not linear.
(3) Prove that the characters of a finite abelian group are all linear.
(4) Define the product of two characters, and show that it is a class function.
(5) Prove
    (a) that the product $\lambda\chi$ of a character χ by a linear character λ is a character, and
    (b) that χ is irreducible if and only if $\lambda\chi$ is irreducible.

Exercise 6.5. Define the dual group of an abelian group, and show that it is indeed a group.