## TRENTO, A.A. 2021/22 GEOMETRY AND LINEAR ALGEBRA EXERCISE SHEET # 11

**Important!** In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

*Exercise* 11.1. State the spectral theorem.

Exercise 11.2. Consider the matrix

$$A = \begin{bmatrix} -2 & 1 & 1\\ 1 & -2 & 1\\ 1 & 1 & -2 \end{bmatrix}.$$

- (1) Can we tell at once whether the eigenvalues of A are real? (And in case, why.)
- (2) Can we tell at once whether A can be diagonalised? (And in case, why.)
- (3) Find the eigenvalues of A.
- (4) If the matrix can be diagonalised, find a basis of eigenvectors.