TRENTO, A.A. 2021/22
GEOMETRY AND LINEAR ALGEBRA
EXERCISE SHEET # 7

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 7.1. Define a linear function.

Exercise 7.2. Let V be a vector space of dimension 2, and let e_1, e_2 be a base of V.

Consider the linear function $f : V \to V$ given by

$$\begin{array}{rcrcr} f(e_1) & = & e_1 & + & 2e_2, \\ f(e_2) & = & e_1 & - & e_2. \end{array}$$

Write the matrix A of f with respect to the base e_1, e_2.

Consider the vectors

$$\begin{array}{rcrcr} g_1 & = & e_1 & + & 2e_2, \\ g_2 & = & 3e_1 & - & e_2. \end{array}$$

(1) Show that g_1, g_2 are a base of V,
(2) write e_1, e_2 as linear combinations of g_1, g_2 (if you spell this out, you will see it is a matter of solving a system of linear equations), and
(3) write the matrix B of f with respect to the base g_1, g_2.

Exercise 7.3. Let V be a vector space of dimension 3, and let e_1, e_2, e_3 be a base of V.

Consider the linear function $f : V \to V$ given by

$$\begin{array}{rcrcrcr} f(e_1) & = & e_1 & + & e_2 & & \\ f(e_2) & = & & & e_2 & + & e_3 \\ f(e_3) & = & e_1 & & & - & e_3 \end{array}$$

Write the matrix A of f with respect to the base e_1, e_2, e_3.

Consider the vectors

$$\begin{array}{rcrcrcr} g_1 & = & e_1, & & & & \\ g_2 & = & e_1 & - & e_2 & - & e_3, \\ g_3 & = & & & & & e_3. \end{array}$$

(1) Show that g_1, g_2, g_3 are a base of V,
(2) write e_1, e_2, e_3 as linear combinations of g_1, g_2, g_3, and
(3) write the matrix B of f with respect to the base g_1, g_2, g_3.