TRENTO, A.A. 2021/22
GEOMETRY AND LINEAR ALGEBRA
EXERCISE SHEET # 6

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 6.1. Say whether each of the following matrices A are invertible or not (in other words, compute the determinant), and in case compute an inverse, that is, a matrix B such that $AB = I = BA$, where I is an identity matrix of a suitable size.

$$[7], \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 0 \\ 7 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 0 & 0 \\ -1 & -1 & -1 & 2 \\ 1 & -1 & 0 & -2 \end{bmatrix}.$$

Exercise 6.2. Consider the following subset of $\mathbf{R}^4$:

$$S = \{(1, 1, 0, 0),\ (1, 0, 1, 0)\}.$$

Check that the vectors of S are linearly independent and extend S to a basis of $\mathbf{R}^4$.

Exercise 6.3. Consider the following subset of $\mathbf{R}^4$:

$$S = \{(1, 1, 0, 0),\ (1, 0, 1, 1),\ (2, 1, 1, 1),\ (0, 1, 1, 0),\ (1, 1, 2, 1)\}.$$

Let $V = \operatorname{span}(S)$. Find a basis of V.