

**TRENTO, A.A. 2021/22**  
**GEOMETRY AND LINEAR ALGEBRA**  
**EXERCISE SHEET # 4**

**Important!** In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

**Important!** Reduce all of the complete matrices in the following exercises to RREF forms.

*Exercise 4.1.*

- (1) Define the (row) rank of a matrix.
- (2) State the Theorem of Rouché-Capelli.

*Exercise 4.2.* State whether the system in the unknowns  $x_1, x_2$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 = 2 \\ 2x_1 + 2x_2 = 3 \end{cases}$$

*Exercise 4.3.* State whether the system in the unknowns  $x_1, x_2$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 = 2 \\ 2x_1 + 2x_2 = 4 \end{cases}$$

*Exercise 4.4.* State whether the system in the unknowns  $x_1, x_2, x_3$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 = 2 \\ x_1 + 2x_2 = 3 \end{cases}$$

*Exercise 4.5.* State whether the system in the unknowns  $x_1, x_2, x_3$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + 2x_2 + 3x_3 = 0 \\ x_1 - x_2 - x_3 = 0 \end{cases}$$

*Exercise 4.6.* State whether the system in the unknowns  $x_1, x_2, x_3$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + 2x_2 + 3x_3 = 0 \\ x_1 - x_2 - 3x_3 = 0 \end{cases}$$

*Exercise 4.7.* State whether the system in the unknowns  $x_1, x_2, x_3$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + 2x_2 + 3x_3 = 2 \\ x_1 - x_2 - x_3 = 3 \end{cases}$$

*Exercise 4.8.* State whether the system in the unknowns  $x_1, x_2, x_3$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + 2x_2 + 3x_3 = 2 \\ x_1 - x_2 - 3x_3 = 3 \end{cases}$$

*Exercise 4.9.* State whether the system in the unknowns  $x_1, x_2, x_3, x_4$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 - x_3 - 3x_4 = 1 \\ 3x_1 - 3x_2 - x_3 - 5x_4 = 0. \end{cases}$$

*Exercise 4.10.* State whether the system in the unknowns  $x_1, x_2, x_3, x_4$  has solutions, and in case find them all.

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 - x_3 - 3x_4 = 1 \\ 3x_1 - 3x_2 - x_3 - 5x_4 = 3. \end{cases}$$