TRENTO, A.A. 2021/22
GEOMETRY AND LINEAR ALGEBRA
EXERCISE SHEET # 3

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 3.1. Consider the following matrices

$$b_1 = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, b_2 = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 5 \end{bmatrix}, b_3 = \begin{bmatrix} -1 & -2 \\ 3 & 5 \\ 1 & 1 \end{bmatrix}, b_4 = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}.$$

Indicate which of the sixteen products $b_i \cdot b_j$ are allowable (I mean, which products make sense), and compute them.

Exercise 3.2. Consider the matrices

$$a = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Compute $a \cdot b$ and $b \cdot a$.

Deduce the following

- when multiplying matrices a and b, it might well happen that $a \cdot b \neq b \cdot a$ even when both products are allowable, and
- the product of two non-zero matrices (meaning that not all if their entries are zero) might well be the zero matrix.

Warning! The next exercises may refer to material not yet seen in the lectures.

Exercise 3.3. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}.$$

Compute

$$S_{13}A, \quad S_{24}A, \quad E_{14}(-1)A, \quad E_{21}(-2)A, \quad D_2(1/2)A, \quad D_4(-1)A.$$

Recall that the elementary row operations are

(1) Exchanging two rows. (We will denote by S_{ij} the operation which exhanges the i-th row and the j-th one.)
(2) Adding to a row a scalar multiple of another. (We will denote by $E_{ij}(c)$ the operation that replaces the i-th row with the sum of the i-th row and the multiple by the scalar c of the j-th row.)

(3) Multiplying a row by a non-zero scalar. (We will denote by $D_i(c)$ the operation that multiplies the i-th row by the scalar $c \neq 0$.)

Exercise 3.4. Find the solutions of the following systems in x, y, z, by putting in RREF form the associated matrices, and try and give a geometric interpretation.

$$\begin{cases} x+y+z=0 \\ x-y+2z=0 \end{cases} \qquad \begin{cases} x+y+z=0 \\ x-y+2z=0 \\ 3x-y+5z=0 \end{cases} \qquad \begin{cases} x+y+z=0 \\ x-y+2z=0 \\ 3x-y+4z=0 \end{cases}$$

Exercise 3.5. Redo the examples of reduction in the notes.