TRENTO, A.A. 2021/22
GEOMETRY AND LINEAR ALGEBRA
EXERCISE SHEET # 2

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 2.1. Find a parametric equation of the line in the plane $\mathbf{R}^2$ described by the implicit (Cartesian) equation $x + 4y = 7$ in the unknowns x, y.

Exercise 2.2. Find a parametric equation of the line of the line in the plane $\mathbf{R}^2$ described by the implicit (Cartesian) equation $4y = 7$ in the unknowns x, y.

Exercise 2.3. Find a Cartesian equation of the line in the plane $\mathbf{R}^2$ described by the parametric equation $(x, y) = (3, 4) + t(1, 5)$.

Exercise 2.4. Find a Cartesian equation of the line in the plane $\mathbf{R}^2$ described by the parametric equation $(x, y) = (3, 4) + t(5, 0)$.

Exercise 2.5. Discuss the following systems of equations in x, y from the point of view of intersections of lines. If one of the systems admits a single solution, find it.

$$\begin{cases} x + y = 1 \\ x + 2y = 0 \end{cases} \qquad \begin{cases} x + y = 1 \\ 2x + 2y = 0 \end{cases} \qquad \begin{cases} x + y = 1 \\ 2x + 2y = 2 \end{cases}$$

Exercise 2.6.

(1) Define the scalar product of two vectors in the plane.
(2) Let $v, w \in \mathbf{R}^2$ be two vectors of length one. Show that the scalar product v, w equals the cosine of the angle between the two vectors.
(3) (Optional) Let $v, w \in \mathbf{R}^2$ be two vectors. Show that the scalar product v, w equals $\|v\| \cdot \|w\| \cdot \cos(\alpha)$, where α the cosine of the angle between the two vectors.

Exercise 2.7. Let $p = (1, 2), q = (2, 3)$. Find parametric and Cartesian equations of the line through p and q.

Exercise 2.8. Consider the line $\mathfrak{l}$ given by the parametric equations

$$\begin{cases} x = 1 + 2t \\ y = 3 + 5t \end{cases}$$

. Find the orthogonal projection of the point $p = (0, 0)$ onto $\mathfrak{l}$, that is, the intersection of $\mathfrak{l}$ with the line $\mathfrak{r}$ passing through the point p, and orthogonal to $\mathfrak{l}$.