TRENTO, A.A. 2021/22
GEOMETRY AND LINEAR ALGEBRA
EXERCISE SHEET # 1

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 1.1. Discuss the solutions x of the equation $ax = b$, where a, b are real numbers.

(HINT: You should say under which conditions on a, b the equation has solutions, and then how many; and say under which conditions on a, b the equation has no solutions.)

Exercise 1.2. Discuss the solutions x, y of the equation

$$ax + by = c. \tag{1}$$

In detail,

(1) Discuss the case $a = b = 0$.
(2) Show how to find the solutions when $a \neq 0$.
(3) Show how to find the solutions when $b \neq 0$.
(4) Show that if a, b are both non-zero, and if x_0, y_0 is a solution of (1), then the solutions of (1) are exactly the pairs x, y of the form

$$\begin{cases} x = -bt + x_0 \\ y = at + y_0 \end{cases}$$

where t is an arbitrary real number. (HINT: We are going to discuss this point in the lectures, but try and do it yourself.)

Exercise 1.3.

(1) Give the formula for the length $\|v\|$ of a vector v in $\mathbf{R}^2$.
(2) Define the multiplication tv of a vector v by a scalar t (i.e. a real number).
(3) Show that if v is a vector and t is a scalar, then $\|tv\| = |\,t\,| \cdot \|v\|$, where $|\,t\,|$ denotes the absolute value of v.
(4) Show that if $v \neq (0, 0) = O$ is a vector, and $t0$ is a number, then the vector tv lies on the line from O to v.